The corresponding matrix of this transformation is

$$\begin{pmatrix} 1 & -\lambda & \lambda^2 & \cdots & (-\lambda)^{\alpha-1} \\ 0 & 1 & -2\lambda & \cdots & \vdots \\ 0 & 0 & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

which is an upper triangular matrix with non-zero determinant, hence invertible. Thus we can write every element in the basis $\{1, \overline{x}, \overline{x}^2, \cdots, \overline{x}^{\alpha-1}\}$ in terms of $\{1, \overline{x}-\lambda,\ (\overline{x}-\lambda)^2, \cdots,\ (\overline{x}-\lambda)^{\alpha-1}\}$. Therefore the second set of elements is also a basis for the transformation T.

Apply T to this basis to write the matrix of T with respect to it:

$$\begin{array}{ccccc} T(1) & = & \overline{x}\cdot 1 & = & \lambda\cdot 1 + 1\cdot(\overline{x}-\lambda) \\ T(\overline{x}-\lambda) & = & \overline{x}\cdot(\overline{x}-\lambda) & = & \lambda\cdot(\overline{x}-\lambda) + 1\cdot(\overline{x}-\lambda)^2 \\ & & & \vdots & \\ T((\overline{x}-\lambda)^{\alpha-1}) & = & \overline{x}\cdot(\overline{x}-\lambda)^{\alpha-1} & = & \lambda\cdot(\overline{x}-\lambda)^{\alpha-1} + (\overline{x}-\lambda)^{\alpha} = \lambda\cdot(\overline{x}-\lambda)^{\alpha-1} \end{array}$$

where the $(\overline{x}-\lambda)^{\alpha}$ vanishes since it is in the quotient. Therefore the matrix is

$$\begin{pmatrix} \lambda & & & & \\ 1 & \lambda & & & \\ & \ddots & \ddots & & \\ & & 1 & \lambda & \\ & & & 1 & \lambda \end{pmatrix}$$

where all the blank entries are zeroes.

Definition. The above matrix is called the **Jordan Block matrix of size k with Eigenvalue λ.**

Back to V:

$$V \cong {}^{\mathcal{F}[x]}\!\big/_{\langle (x-\lambda_1)^{\alpha_1}\rangle} \oplus {}^{\mathcal{F}[x]}\!\big/_{\langle (x-\lambda_2)^{\alpha_2}\rangle} \oplus \cdots \oplus {}^{\mathcal{F}[x]}\!\big/_{\langle (x-\lambda_t)^{\alpha_t}\rangle}$$

The new matrix of V with respect to the new bases of each direct summand is a block diagonal matrix as below: