Does h makes the diagram commute? Take $M \in |\mathbb{Q}|$.

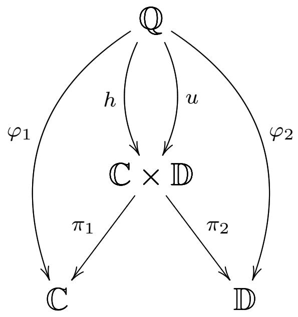
$$\begin{aligned}(\pi_1 \circ h)(M) &= \pi_1(h(M)) \\ &= \pi_1(\varphi_1(M), \varphi_2(M)) \\ &= \varphi_1(M)\end{aligned}$$

Take $f \in \mathrm{Arr}(\mathbb{Q})$.

$$\begin{aligned}(\pi_1 \circ h)(f) &= \pi_1(h(f)) \\ &= \pi_1(\varphi_1(f), \varphi_2(f)) \\ &= \varphi_1(f)\end{aligned}$$

Therefore $\pi_1 \circ h = \varphi_1$ since they act the same on objects and arrows, and, similarly, $\pi_2 \circ h = \varphi_2$. Therefore the diagram commutes.

Is h unique? Suppose $\exists\ u$ with



such that u also makes the diagram commute. Take $M \in |\mathbb{Q}|$. Since $\varphi_i(M) = \pi_i(u(M))$, thinking of π_i's as the "projections", we can see that $u(M) = (\varphi_1(M), \varphi_2(M))$. But $h(M) = (\varphi_1(M), \varphi_2(M))$, so u and h are equal on all the objects of $\mathbb{Q}$.

Take $f \in \mathrm{Arr}(\mathbb{Q})$. Since $\varphi_i(f) = \pi_i(u(f))$, thinking of π_i's as the "projections", we can see that $u(f) = (\varphi_1(f), \varphi_2(f))$. But $h(f) = (\varphi_1(f), \varphi_2(f))$, so u and h are equal on all the arrows of $\mathbb{Q}$.

Hence $u = h$ and so h is indeed unique.