Consider the following indicator variables:

$$X_i = \begin{cases} 1 & \text{if the length of } I_i \text{ is greater than } \lambda \\ 0 & \text{else} \end{cases}$$

Investigating some examples, we realise that

$$\begin{aligned} C - 1 &= \sum_{i=1}^{n-1} I_i \\ C &= 1 + \sum_{i=1}^{n-1} I_i \end{aligned}$$

Thus, if $C = r$,

$$\begin{aligned} C = r &\implies 1 + \sum_{i=1}^{n-1} I_i = r \\ &\implies \sum_{i=1}^{n-1} I_i = r - 1 \end{aligned}$$

So, the event that $C = r$ is the same as the event that there are exactly $r-1$ intervals of length greater than λ. Hence

$$\begin{aligned} \mathbb{P}(C = r) &= \binom{n-1}{r-1} \mathbb{P}\left(\left\{\bigcap_{i=1}^{r-1} X_{\sigma_i} = 1\right\} \bigcap \left\{\bigcap_{i=r}^{n-1} X_{\sigma_i} = 0\right\}\right) \\ &= \binom{n-1}{r-1} \mathbb{P}\left(\left\{\bigcap_{i=1}^{r-1} |I_{\sigma_i}| > \lambda\right\} \bigcap \left\{\bigcap_{i=r}^{n-1} |I_{\sigma_i}| \leq \lambda\right\}\right) \\ &= \binom{n-1}{r-1} \mathbb{P}\left(\bigcap_{i=1}^{r-1} |I_{\sigma_i}| > \lambda\right) \bigcap \mathbb{P}\left(\bigcap_{i=r}^{n-1} |I_{\sigma_i}| \leq \lambda\right) \\ &= \binom{n-1}{r-1} \mathbb{P}(|I_{\sigma_i}| > \lambda)^{r-1} \bigcap \mathbb{P}(|I_{\sigma_i}| \leq \lambda)^{n-r} \end{aligned}$$

Look at each of the factors. Let x_{σ_i}, $x_{\sigma_{i+1}}$ be the two points of the random geometric graph that form the endpoints of I_{σ_i}. Then the length of I_{σ_i} being less than or equal to λ or greater than λ is precisely the event that $x_{\sigma_i} x_{\sigma_{i+1}}$ is an edge of the random geometric graph or not. By Theorem 2.5, we then get the following equation: