5.3 Tweedledum-Tweedledee on ab*3*

Recall the Tweedledum-Tweedledee strategy from Definition 1.3.4: Suppose Left and Right are playing in the position $\xi+\overline{\xi}$ with Left moving first. Either Left has no move available, or, without loss of generality, she can move to $\xi^L+\overline{\xi}$. Right can respond by moving to $\xi^L+\overline{\xi}^R$. However, $\overline{\xi}^R=\overline{\xi^L}$, so Right has moved to $\xi^L+\overline{\xi^L}$. Now either Left has no move available, or Left has a move available, to which Right can respond by making the mirror-image move in the opposite component. This strategy ensures that the second player makes the final move. In normal play, it is a winning strategy for the second player in the position $\xi+\overline{\xi}$. In misère play, it is a particularly poor strategy for the second player, as not only will it guarantee his loss, we cannot even say whether the outcome of $\xi+\overline{\xi}$ is $\mathcal{N}$ or $\mathcal{P}$ for arbitrary ξ (Proposition 5.2.1). However, as Proposition 5.2.4 showed, if $\xi\in c\ell\,(\mathrm{ab}\mathit{3})$, then $o^-(\xi+\overline{\xi})=\mathcal{N}$. Using this fact, we are able to construct a strategy for $\xi+\overline{\xi}$ if ξ is ab*3* which mimics Tweedledum-Tweedledee strategy for normal play. It is important to note that this strategy gives a win for the next player to move, not the previous player as the Tweedledum-Tweedledee strategy for normal play does.

Construction 5.3.1 (A Tweedledum-Tweedledee type strategy for $c\ell\,(\mathrm{ab}\mathit{3})$)**.** *Take an arbitrary element of* $c\ell\,(\mathrm{ab}\mathit{3})$*, say* $\sum_{i=1}^{n}\xi_i$ *where each* ξ_i *is* ab*3. Then we are looking to construct a winning strategy for Left moving first in*

$$\sum_{i=1}^{n}\xi_i+\overline{\sum_{i=1}^{n}\xi_i}=\sum_{i=1}^{n}(\xi_i+\overline{\xi_i}).$$

The overview of the algorithm is as such:

1. *Consider the sum as follows:*

$$\xi_1+\overline{\xi_1}+\sum_{i=2}^{n}\xi_i+\overline{\sum_{i=2}^{n}\xi_i}$$

2. *Left's first move is in* $\xi_1+\overline{\xi_1}$*.*

3. *If Right responds by playing in* $(\xi_1+\overline{\xi_1})^L$*, then so does Left.*