30th Foundational Methods in Computer Science

Mount Allison University

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Foundational Methods in Computer Science is an annual workshop that brings together researchers in theoretical computer science and category theory. Past workshops have had discussions on areas such as quantum programming languages, restriction categories, database design, and the differential and resource logics. They have been held at Colgate, Dalhousie, Kananaskis (U of Calgary), Mount Allison, Ottawa, UBC, Spokane, and Portland. The workshop is informal and interdisciplinary.

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Tutorials

Categories of resistors

Robin * Cockett

A classical Electrical Engineering problem is to determine whether two networks of resisters are equivalent. The standard solution is to use a process of eliminating internal nodes (the star/mesh transformation). This may be seen as a rewriting procedure, or it may, as is common in the EE literature, be organized into a matrix problem which can be solved by a Gaussian elimination procedure. This approach essentially works for resistances taken from the positive reals: furthermore, the technique secretly uses negatives. However, despite this it cannot actually handle negative resistances. Negative resistances, while counter-intuitive, actually arise in stabilizer quantum mechanics where networks of "resistors" over finite fields are occur.

The rewriting approach is more general and, at first blush, works for any positive division rig. We shall discuss how the rewriting theory can be modified to handle negatives (which we think is novel).

In the talk we shall start by introducing categories of resistors. These are "hypergraph categories" whose maps are generated by resistors. Resistors are then self-dual maps satisfying certain formal properties (including an infinite family of star-mesh equalities). The problem of network equivalence is, thus, the decision problem for maps in these categories. The fact that there is a terminating and confluent rewriting system not only solves the decision problem (up to certain equalities) but also has the effect of guaranteeing, very basically, the associativity of the composition.

Work with: Priyaa Srinivasan and Amolak Kalra

Cartesian double categories

MICHAEL * LAMBERT

Cartesian double categories are ubiquitous. I hope in this talk to convince you that they are both useful and interesting. After surveying a bit of history of their development, I will discuss a number of examples, both new and old. The main point of these talks will be eventually to look in particular at the example of monoids and modules in a suitably structured double category. This archetypical example will be seen to embrace many of the former specialized examples. The main result is that under fairly general conditions, such monoids and modules do in fact form a cartesian double category which is also an equipment. Time permitting, we'll close by looking at some applications of cartesian double categories to data migration.

Double Fibrations, Double Grothendieck and Double Colimits

DORETTE * PRONK

Classically, the Grothendieck construction, or the category of elements, of a pseudo functor $F\colon \mathbf{D} \to \mathbf{Cat}$ (also called the indexing functor) has two properties that define it (up to equivalence) (see [AGV71]):

(A) it is the object part of the adjoint equivalence between the category of such pseudo functors, also called indexing functors, and the category of fibrations over $\mathbf{D}$;

(B) it is the lax colimit for the diagram defined by F.

These two results have been shown to hold for suitably defined Grothendieck constructions for bicategories and ∞-categories and has been conjectured for weak n-categories. So when one is interested in fibrations between double categories we want to look for a suitable Grothendieck construction. And the next question will then be how they may also be seen as colimits. The latter question is particularly relevant when we want to consider maps between the double categories that are given in terms of an elements construction.

We begin the journey into double categories with a suitable notion of double fibration, namely that of an pseudo-category object [Mar06] in a suitable 2-category of fibrations. In the tutorial I will discuss some of its properties and show how it generalizes several previously studied concepts such as monoidal fibrations [MV20]. In order to obtain these fibrations through an elements construction, we consider indexing functors into the double 2-category $\mathbb{S}$pan(Cat) (and we will work with all lax functors). Note that a lax double functor from the terminal double category into $\mathbb{S}$pan(Cat) corresponds to a double category. The Grothendieck/elements construction for this type of indexing functors can be viewed as a Grothendieck construction that is done in two layers and produces indeed a category object in a suitable category of fibrations; i.e., a double fibration. This construction also generalizes the one given in [MV20]. Furthermore, structured cospans and decorated cospans [Pat23], as well as the double Grothendieck construction for double categories of open dynamical systems introduced in [Jaz21], can be viewed as special cases of this construction.

Viewing this construction as a suitable type of colimit of a diagram of double categories would give us a natural notion of morphism between these double categories of elements. To achieve this, we will show that pseudo double functors $F\colon \mathbb{D} \to \mathbb{S}\text{pan(Cat)}$ correspond to pseudo double functors $\tilde{F}\colon \mathbb{D} \to \mathbb{P}\text{rof(Cat)}$, where $\mathbb{P}$rof(DblCat) is the double category of double categories with internal profunctors in Cat as pro-arrows and pseudo double functors as arrows. As part of work in progress I will indicate how the double category of elements for F forms a lax colimit for the diagram defined by $\tilde{F}$ (and discuss the universal property of these colimits). This suggests that the appropriate notion of morphism between decorated cospans is that of a suitable kind of modules (internal profunctors in **Cat**).
A significant part of the content of this talk is joint work with Geoff Cruttwell, Michael Lambert and Martin Szyld in [LCS22]. For those interested in working on this project, I welcome you to join

me in a research discussion session to strategize and start working on the proofs for a joint paper on the last part of the talk.

Bibliography

[AGV71] M. Artin, A. Grothendieck and J. Verdier. "Théorie de Topos et Cohomologie Étale des Schémas I". In: *Lecture Notes in Mathematics* 269 (1971).

[Jaz21] D. Jaz Myers. "Double Categories of Open Dynamical Systems". In: *Electronic Proceedings in Theoretical Computer Science* 333 (2021), pp. 154–167.

[LCS22] M.J. Lambert, G. S. H. Cruttwell and M. Szyld. "Double Fibrations". In: *Theory and Applications of Categories* 38 (35) (2022), pp. 1326–1394.

[Mar06] N. Martins-Ferreira. "Pseudo-Categories". In: *Journal of Homotopy and Related Structures* 1 (2006), pp. 47–78.

[MV20] J. Moeller and C. Vasilakopoulou. "Monoidal Grothendieck Construction". In: *Theory and Applications of Categories* 35 (31) (2020), pp. 1159–1207.

[Pat23] Evan Patterson. "Structured and decorated cospans from the viewpoint of double category theory". In: (2023).

Introduction to the stabilizer calculus

PETER * SELINGER

The stabilizer calculus is a very useful tool in quantum computing, but it also has its own mathematical attractiveness. It first came to widespead notice in 1998, when Gottesman and Knill used the stabilizer formalism to prove that a certain class of quantum gates can be efficiently simulated on a classical computer (and therefore does not give a quantum speedup). The stabilizer formalism is also useful in the design of quantum error correcting codes (which would be basically incomprehensible without it). This goes back to Gottesman's 1997 Ph.D. thesis. More recently, the stabilizer formalism has also been used to understand graph states, lattice surgery, and many other things will be essential to the design of large-scale future quantum computers. Category theorists will be happy to learn that it also ties in with the ZX-calculus, another versatile tool for reasoning about all things quantum. One might say that a good formalism is one that makes a complicated thing simple (other examples that come to mind are decimal notation, fractions, parentheses, Leibniz's notation for derivatives, and categorical diagrams). By that standard, the stabilizer calculus is a good formalism. I will use the tutorial to explain the basics, and then do as much or little as we have time for.

Message passing logic for categorical quantum mechanics

Priyaa * Srinivasan

Cockett and Pastro introduced two-tiered 'Message Passing Logic' in order to develop a type theory for concurrent programs with message passing as the communication primitive [CP09]. The motivation was to allow one to guarantee certain formal properties of concurrent programs such as deadlock and livelock avoidance, which is not possible using the current programming technologies. Current programming languages lack such features because concurrency constructs are embedded in the sequential core without a formal underpinning. The categorical logic behind the machinery introduced by Cockett and Pastro is based on monoidal categories acting on linearly distributive categories with the proof theory given by multi categories acting on poly categories, and is called a linear actegory. Chad Nester introduced resource transducers for concurrent process histories which is a toy model of linear actegories. In this tutorial, I shall introduce Cockett and Pastro's message passing logic, Nester's resource transducers and present preliminary results on the connections of these structures to categorical quantum mechanics. The aim is to explore if one can develop categorical quantum message passing by lifting these structures to dagger isomix categories introduced in my thesis.

Bibliography

[CCS21] Robin Cockett, Cole Comfort and Priyaa Srinivasan. "Dagger linear logic for categorical quantum mechanics". In: *Logical Methods in Computer Science* Volume 17, Issue 4 (Nov. 2021).

[CP09] J.R.B. Cockett and Craig Pastro. "The logic of message-passing". In: *Science of Computer Programming* 74.8 (2009). Special Issue on Mathematics of Program Construction (MPC 2006), pp. 498–533.

[Nes23] Chad Nester. "Concurrent Process Histories and Resource Transducers". In: (2023).

Contributed talks

Profunctors and Promonoidal Categories

Amélie * Comtois

We introduce profunctors: a generalization of functors. Much like how a function is a special kind of relation, a functor is a special kind of profunctor. We will explore this analogy and its consequences in logic. Then, we consider promonoidal categories, which are like monoidal categories but where we replace functors with profunctors. Finally, we explore the monoidal structure of the functor category on a monoidal category and the analogous case on a promonoidal category.

Introduction to CaMPL: part 1

Saina * Daneshmandjahromi

The **C**ategorical **M**essage **P**assing **L**anguage (CaMPL) is a statically typed functional style concurrent programming language. It is a "two-tiered" language with a sequential language integrated into a concurrent language. CaMPL features constructs for both sequential and concurrent computation. I will introduce the sequential constructs, including data declarations, codata declarations, and function definitions. These constructs have many ideas in common with traditional functional languages. I will also discuss the concurrent side of CaMPL: this part uses message passing between two processes along exactly one channel according to a protocol. I will discuss the basic concurrent features including processes, and their commands: plug, close, halt, get, put, hcase, hput. I will aim to give an intuition for writing CMPL programs and demonstrate some example programs written in the language.

Developing a module system for CaMPL

Braden * Foxcroft

CaMPL is a categorical message-passing language which uses compile-time type-checking to prevent deadlocks and livelocks. This talk will present a high-level discussion about implementing a module system for CaMPL, considering alternative implementations and the benefits and drawbacks of each.

Towards an induction principle for nested data types

Frank * Fu

A well-known problem in the theory of dependent types is how to handle so-called nested data types. These data types are difficult to program and to reason about in total dependently typed languages such as Agda and Coq. In particular, it is not easy to derive a canonical induction principle for such types. Working towards a solution to this problem, we introduce dependently typed folds for nested data types. Using the nested data type Bush as a guiding example, we show how to derive its dependently typed fold and induction principle. We also discuss the relationship between dependently typed folds and the more traditional higher-order folds.

Quantale-valued relations and cartesian quantaloids

Rose * Kudzman-Blais

Locally posetal *cartesian bicategories* were introduced by Carboni and Walters to generalize Rel, the bicategory of sets and relations, and more generally to provide a theory of relations. A binary relation may be viewed as a function into the two-element *quantale* {true, false}. In order to model situations with quantitative information, we can consider the bicategory of sets and quantale-valued relations $Q\text{−}\mathsf{Rel}$, standard in *monoidal topology*. Provided that the quantale is in fact a locale, $Q\text{−}\mathsf{Rel}$ is a cartesian bicategory.

$Q\text{−}\mathsf{Rel}$ is furthermore a *quantaloid*, the bicategorical equivalent of the quantale. Pitts studied quantaloids that are cartesian bicategories and introduced *cartesian quantaloids*, which are the appropriate bicategorical version of locales. One consequence of these ideas is that $Q\text{−}\mathsf{Rel}$ is a cartesian bicategory if and only if Q is a locale.

Niefield and Rosenthal studied how to construct locales from quantales by taking quotients via certain closure operations, known as localic quantic nuclei. This construction yields a left adjoint to the inclusion of the category of locales in to the category of quantales. The notion of quantic nucleus has been adapted by Rosenthal to the quantaloidal context and is known as a *quantaloidal nucleus*. We show that a quantic nucleus on Q can be extended to a quantaloidal nucleus on $Q\text{−}\mathsf{Rel}$ and that, if it was localic on Q, then the resulting quotient quantaloid is cartesian.

The main objective of this work is to generalize this relationship between quantales and locales to the setting of quantaloids and cartesian bicategories, with $Q\text{−}\mathsf{Rel}$ as the motivating example. To this end, we define *pre-cartesian* quantaloids which capture the necessary structure of $Q\text{−}\mathsf{Rel}$. It follows that a quotient of a pre-cartesian quantaloid is cartesian if and only if the quantaloidal nucleus satisfies two conditions.

Algebraic Deformation and tangent categories

Marcello * Lanfranchi

The fundamental idea of deformation theory is to slightly modify an algebraic or a geometrical object so that the new one is still of the same kind of the original one, but its properties are different.

For example, the deformation of an affine scheme corresponds to slightly changing the relations that define the corresponding coordinate ring.

In this talk, we would like to present some interesting connections between algebraic deformation theory and tangent categories. We will show how to relate infinitesimal deformations of algebras to vector fields of a suitable tangent category. To explore this idea, we will make use of the theory of operads, by showing that the category (and also its opposite one) of all algebraic operads over a fixed commutative and unital ring R, form a tangent category, naturally included in the tangent category of tangent monads over a tangent category.

As part of my PhD research, this work is in collaboration with my supervisors Dorette Pronk and Geoffrey Cruttwell.

I would like also to thank Geoff Vooys for the useful discussions I had with him on deformation theory.

Classical Distributive Restriction Categories

Jean-Simon Pacaud * Lemay

In the category of sets and partial functions, the Cartesian product $\times$ is not a categorical product. Instead the categorical product is given by $A + B + A \times B$, where $+$ is the disjoint union. In this talk, we give a restriction category explanation of why this is the case. We will show that in a distributive restriction category, $A + B + A \times B$ is a product if and only if the restriction category is classical, that is, has joins and relative complements.
This is joint work with Robin Cockett.

Semantics for Non-Determinism in Categorical Message Passing Language by Sup-Lattice Enrichment

Alexanna * Little

Categorical Message Passing Language (CaMPL) is a functional concurrent programming language, and the semantics for its programming features are defined by categorical structures. In this talk, we explore the categorical semantics for programming features in CaMPL, especially "races" which introduce non-determinism. The semantics for non-determinism is given by sup-lattice enrichment, and we will discuss why this gives an accurate model of non-determinism and how this is achieved.

Symmetric multicategories arising from base-valued enriched functors

Rory * Lucyshyn-Wright

Multicategories, also called coloured operads, were introduced by Lambek in his categorical studies of logic and linguistics. A multicategory is much like a category except that each morphism has not just a single object as its domain but rather a list of objects, while a symmetric multicategory is equipped also with certain actions of the symmetric groups that have the effect of permuting inputs. For example, the category of real vector spaces underlies a symmetric multicategory in which the morphisms are multilinear maps. More generally, it is well known that all commutative monadic categories C over Set and all topological categories C over Set underlie symmetric multicategories whose morphisms are, in a suitable sense, C-morphisms in each variable separately.

In this talk, we provide a general result that gives rise to many examples of symmetric multicategories, both new and old, by generalizing the notion of "morphism in each variable separately" beyond the above settings of commutative monadic and topological categories—and indeed, well beyond the setting of concrete categories. Given any symmetric multicategory V and any V-enriched category C equipped with a suitably V-enriched functor $G\colon C \to V$, we construct a symmetric multicategory C_G whose underlying ordinary category is the same as that of C. The morphisms in C_G are what we call G-multimorphisms, and they generalize multilinear maps and T-multimorphisms for a commutative monad T, while in our setting G does not even need to be faithful, let alone monadic. For example, this result entails that if C is any category and $G\colon C \to \mathsf{Set}$ is any Set-valued functor, then C underlies a symmetric multicategory C_G whose morphisms are the G-multimorphisms.

An Introduction to Categorical Message Passing Language (Part II)

Melika * Norouzbeygi

Categorical Message Passing Language (CaMPL) is a typed concurrent programming language with full static type inference for both sequential and concurrent programs. The sequential side of CaMPL, is a functional style typed language with data and codata definitions (as will be discussed in more detail in an earlier talk). The concurrent side supports message passing between processes along channels typed by protocols.

In this presentation, I will introduce the key concurrent constructs of CaMPL: Plug, Split, Fork, Id, Neg, and Race. We will demonstrate these constructs with examples. In particular, we will show how the *Memory Cell* can be used to implement mutual exclusion by allowing multiple processes to interact exclusively with a shared memory. We conclude this presentation by discussing some CaMPL syntax improvements.

Initial algebras for topologically enriched multi-sorted algebraic theories

JASON * PARKER

Classically, multi-sorted algebraic theories and their initial algebras have been fundamental in mathematics and computer science. Within the latter field, they have been prominently employed in studying algebraic specification and abstract algebraic datatypes, computational effects, and algebraic databases. In this talk, we present a generalization of multi-sorted algebraic theories from the classical (Set-enriched) context to the context of enrichment in a symmetric monoidal category V that is topological over Set. Prominent examples of such categories V include: various categories of topological spaces; the category of measurable spaces; the categories of models of relational Horn theories without equality, such as the categories of preordered sets and (extended) pseudo-metric spaces; and the categories of quasispaces (a.k.a. concrete sheaves) on concrete sites, which have recently attracted significant interest in the semantics of programming languages.

Given such a category V, we define a notion of V-enriched multi-sorted algebraic theory. We show that every V-enriched multi-sorted algebraic theory T has an underlying classical multi-sorted algebraic theory $|T|$, and that free T-algebras may be obtained as suitable liftings of free $|T|$-algebras. We establish explicit and concrete descriptions of free T-algebras, which have a convenient inductive character when V is cartesian closed. We provide several examples of V-enriched multi-sorted algebraic theories, and we also discuss the close connection between V-enriched multi-sorted algebraic theories and the (presentations of) V-enriched monads and V-enriched algebraic theories studied in recent papers by the author and Rory Lucyshyn-Wright.

Classical Distributive Restriction Categories

Florian * Schwarz

A partial monoid is a set A with a multiplication and a unit, but the multiplication is only defined on a certain subset of pairs. This means the multiplication $m : A \times A \to A$ is a partial map, a map that is only defined on a certain subset of its domain. The category of finite commutative partial monoids is an important ingredient for the construction of tangent infinity categories where a bicategory of spans of partial monoids encodes a weakly commutative ring structure. As partial maps are generalized by restriction categories one can directly generalize the definition of a partial monoid in any restriction category. I will present a certain restriction category B of bags (a.k.a. multisets) with a partial monoid in it. Any partial monoid in any join restriction category X is induced through a functor from B to X which means the partial monoid in B is the generic partial monoid. This characterization of partial monoids is analogous to the concept of a Lawvere theory of an algebraic structure.

String diagrams for symmetric powers

Jean-Baptiste * Vienney

In a symmetric monoidal category Q+-linear category C, ie. a symmetric monoidal category such that every hom-set is a Q+-module, one can define in different equivalent ways a symmetrization of every tensor power $A^{\otimes n}$. It provides the n^{th} symmetric tensor power functor $S^n : C \to C$. These functors are useful in algebra, quantum physics and linear logic.

We show that in a symmetric monoidal Q+-linear category C, the families of objects of the from $(S^n(A))_{n>=1}$ are characterized among the countable families of objects as those which can be equipped with an algebraic structure that we call binomial graded bialgebra.

It provides an elegant string diagrammatic language for symmetric powers.

This characterization is part of broader results: we define the category of binomial graded bialgebras in a symmetric monoidal Q+-linear category along with three other categories of graded objects and show that these four categories are equivalent.

Tangent Ind-Categories

Geoff * Vooys

If C is a category then the ind-category of C, denoted $\mathsf{Ind}(C)$, is the free filtered cocompletion of C. This is an important category in homotopy theory and algebraic geometry, as the free filtered cocompletion is a process which essentially allows one to take "infinitesimal completions" of objects in C. For example, if C is the category of schemes, then $\mathsf{Ind}(C)$ is the category of formal schemes (and hence describes a category of geometric objects formally completed along some closed subobjects). In this talk we will show that if C is a tangent category then there is a natural tangent structure on $\mathsf{Ind}(C)$ and discuss some of the consequences and intuition behind this construction and result.

Generators and relations for 3-qubit Clifford+CS operators

Bian * Xiaoning

We give a presentation by generators and relations of the group of 3-qubit Clifford+CS operators. The proof roughly consists of two parts: (1) applying the Reidemeister-Schreier theorem repeatedly to an earlier result of ours; and (2) the simplification of thousands of relations into 17 relations. Both (1) and (2) have been formally verified in the proof assistant Agda. We also show that the 3-qubit Clifford+CS group, which is of course infinite, is the amalgamated product of three finite subgroups. This result is analogous to the fact that the 1-qubit Clifford+T group is an amalgamated product of two finite subgroups.